A Latent Class Modeling Approach for Differentially Private Synthetic Data for Contingency Tables

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Outline

1. Data privacy
2. Differentially private modeling approach
3. Illustrations with 2016 ACS data
4. Concluding remarks

Privacy and data sharing

- Agencies and companies often seek to share their data.
- Protection of individuals' private information is a must.
- Traditional strategies: disclosure control methods [Hundepool et al., 2012] or releasing synthetic data [Rubin, 1993].
- In recent years, agencies are looking for methods that provide formally quantifiable privacy guarantees, e.g., those that rely on differential privacy.

Problem setup

- Confidential dataset $\boldsymbol{X} = \{X_i = (X_{1i}, \ldots, X_{pi})\}_{i=1}^n$, where X_{ij} is categorical.
- Assume that the agency is willing to release summaries of $\boldsymbol{X}$ denoted by $M(\boldsymbol{X}) = (M_1(\boldsymbol{X}), \ldots, M_T(\boldsymbol{X}))$.
- The goal is to generate a synthetic version of $\boldsymbol{X}$ using $M(\boldsymbol{X})$ and a formally private mechanism.

Illustration with ACS PUMS

- We selected a subset of 10,000 individuals from the 2016 one-year ACS PUMS.
- Each $M_t(\boldsymbol{X}),\ t = 1, \ldots, 10$, denotes a two-way marginal table.

	Age	
Citizenship	0	1
0	11	596
1	443	8950

	Race	
Citizenship	0	1
0	299	308
1	1731	7662

	Sex	
Citizenship	0	1
0	273	334
1	4505	4888

	Income	
Citizenship	0	1
0	294	313
1	2916	6477

	Race	
Age	0	1
0	110	344
1	1920	7626

	Sex	
Age	0	1
0	239	215
1	4539	5007

	Income	
Age	0	1
0	445	9
1	2765	6781

	Sex	
Race	0	1
0	945	1085
1	3833	4137

	Income	
Race	0	1
0	827	1203
1	2382	5587

	Income	
Sex	0	1
0	1281	3497
1	1929	3293

Differential privacy

- Differential privacy is the best known formal privacy framework in use.
- $\mathcal{M}(\boldsymbol{X})$ is a randomized version of $M(\boldsymbol{X})$.

Definition

ϵ-Differential Privacy [Dwork et al, 2006]: A randomized mechanism $\mathcal{M}$ satisfies ϵ-differential privacy if for all data sets $\boldsymbol{X}$ and $\boldsymbol{X}'$ differing on at most one row, and $\mathcal{S} \subseteq \mathrm{Range}(\mathcal{M})$,

$$\frac{\Pr[\mathcal{M}(\boldsymbol{X}) \in S | \boldsymbol{X}]}{\Pr[\mathcal{M}(\boldsymbol{X}') \in S | \boldsymbol{X}']} \leq \exp(\epsilon)\,.$$

Differentially private summary statistics

- $\mathcal{M}(\boldsymbol{X}) = (\mathcal{M}_1(\boldsymbol{X}), \ldots, \mathcal{M}_T(\boldsymbol{X}))$ is a randomized version of $M(\boldsymbol{X}) = (M_1(\boldsymbol{X}), \ldots, M_T(\boldsymbol{X}))$.

Theorem

Geometric Mechanism ***[Ghosh et. al, 2012]****: For $M_t(\boldsymbol{X}) : \mathcal{D} \to \mathbb{Z}^{d_t}$, the mechanism $\mathcal{M}_t$ that adds independently drawn noise from a* two-sided-Geom$(exp\{\frac{-\epsilon_t}{\Delta M_t}\})$ *distribution to each of the d_t terms of $M_t(\boldsymbol{X})$ satisfies ϵ_t-differential privacy.*

- Sensitivity $\Delta M_t = \sup_{\boldsymbol{X},\boldsymbol{X}'} \|M_t(\boldsymbol{X}) - M_t(\boldsymbol{X}')\|_1$.

Illustration with ACS PUMS

- Sequential composition [Mcsherry, 2009]: If each $\mathcal{M}_t$ provides ϵ_t-differential privacy. The sequence of $\mathcal{M}(\boldsymbol{X}) = (\mathcal{M}_1(\boldsymbol{X}), \ldots, \mathcal{M}_T(\boldsymbol{X}))$ provides $(\epsilon = \sum_t \epsilon_t)$-differential privacy. We can use $\epsilon_t = \epsilon / T$.

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Bayesian modeling approach

- The released summary statistic is of the form

$$\mathcal{M}(\boldsymbol{X}) = (M_1(\boldsymbol{X}) + \varepsilon_1, \ldots, M_T(\boldsymbol{X}) + \varepsilon_T).$$

- Some counts based on $\mathcal{M}(\boldsymbol{X})$ will not necessary match.

- Ideal modeling approach:

$$\mathcal{M}_t(\boldsymbol{X}) | M_t(\boldsymbol{X}) \overset{ind}{\sim} \text{two-sided-Geom}_{d_t}\left(M_t(\boldsymbol{X}), exp\left\{\frac{-\epsilon}{\Delta M_t T}\right\}\right),$$
$$M(\boldsymbol{X}) = (M_1(\boldsymbol{X}), \ldots, M_T(\boldsymbol{X}))|\boldsymbol{\theta} \sim p_M(\cdot|\boldsymbol{\theta}),$$
$$\boldsymbol{\theta} \sim p_{\boldsymbol{\theta}}.$$

- It is not easy to characterize $p_M(\cdot|\boldsymbol{\theta})$.

- We know that $M_t(\boldsymbol{X})|\boldsymbol{\theta} \sim \text{Multinomial}_{r_t}(n, P_t(\boldsymbol{\theta}))$.

Bayesian modeling approach using composite likelihood methods

- Proposed modeling approach:

$$\mathcal{M}_t(\boldsymbol{X})|M_t(\boldsymbol{X}) \overset{ind}{\sim} \text{two-sided-Geom}_{d_t}\left(M_t(\boldsymbol{X}), exp\left\{\frac{-\epsilon}{\Delta M_t T}\right\}\right),$$

$$M_t(\boldsymbol{X})|\boldsymbol{\theta} \overset{ind}{\sim} \text{Multinomial}_{d_t}(n, P_t(\boldsymbol{\theta})),\ t = 1, \ldots, T,$$

$$\boldsymbol{\theta} \sim p_{\boldsymbol{\theta}}.$$

- Notice that the probabilities $P_1(\boldsymbol{\theta}), \ldots, P_T(\boldsymbol{\theta})$ are related.

- We can define $P_t(\boldsymbol{\theta})$ by specifying a model for $\boldsymbol{X}|\boldsymbol{\theta}$.

Illustration with ACS PUMS

$M_1(\boldsymbol{X}) =$

	Age	
Citizenship	0	1
0	11	596
1	443	8950

$$P_1(\boldsymbol{\theta}) = \begin{pmatrix} p_{1,(0,0)} & p_{1,(0,1)} \\ p_{1,(1,0)} & p_{1,(1,1)} \end{pmatrix}$$

$M_2(\boldsymbol{X}) =$

	Race	
Citizenship	0	1
0	299	308
1	1731	7662

$$P_2(\boldsymbol{\theta}) = \begin{pmatrix} p_{2,(0,0)} & p_{2,(0,1)} \\ p_{2,(1,0)} & p_{2,(1,1)} \end{pmatrix}$$

- Coherence: $p_{1,(1,0)} + p_{1,(1,1)} = p_{2,(1,0)} + p_{2,(1,1)}$
- We define $P_t(\boldsymbol{\theta})$ by specifying a model for $\boldsymbol{X}|\boldsymbol{\theta}$.

Modeling $\boldsymbol{X}|\boldsymbol{\theta}$

- We use the following mixture model [Dunson and Xing 2009]:

$$
\begin{aligned}
X_{ij}|z_i, \{\Psi_h^{(j)}\}_{h=1}^{\infty} &\overset{ind}{\sim} \mathit{Multinomial}\{1, \Psi_{z_i 1}^{(j)}, \ldots, \Psi_{z_i d_j}^{(j)}\}, \\
z_i|\{\pi_h\}_{h=1}^{\infty} &\overset{ind}{\sim} \mathit{Discrete}\{(1, \ldots, \infty), (\pi_1, \ldots, \pi_\infty)\}, \\
\pi_h &= V_h \prod_{l<h}(1 - V_l), \quad V_h \sim \beta(1, \alpha), \\
\Psi_h^{(j)} &\sim \mathit{Dirichlet}(a_{j1}, \ldots, a_{jd_j}),
\end{aligned}
$$

where $\boldsymbol{\theta} = \left(\boldsymbol{\pi}_k = \{\pi_h\}_{h=1}^{k},\ \boldsymbol{\Psi}_k = \{\Psi_h^{(j)}\}_{h=1,j=1}^{k,p}\right)$.

Defining $P_1(\boldsymbol{\theta}), \ldots, P_T(\boldsymbol{\theta})$

- If $M_1(\boldsymbol{X})$ is the contingency table of the first two variables, then

$$P_1(\boldsymbol{\theta}) = \begin{pmatrix} p_{1,(0,0)} & p_{1,(0,1)} \\ p_{1,(1,0)} & p_{1,(1,1)} \end{pmatrix}$$

where, e.g.,

$$p_{1,(0,0)} = Pr(X_{\cdot 1} = 0, X_{\cdot 2} = 0|\boldsymbol{\theta}) = \sum_{h=1}^{k} \pi_h \Psi_{h0}^{(1)} \Psi_{h0}^{(2)} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{l=0}^{1} \Psi_{hi}^{(3)} \Psi_{hj}^{(4)} \Psi_{hk}^{(5)}.$$

Bayesian modeling approach and inference

- Proposed approach:

$$
\begin{aligned}
\mathcal{M}_t(\boldsymbol{X})|M_t(\boldsymbol{X}) &\overset{ind}{\sim} \text{two-sided-Geom}_{d_t}\left(M_t(\boldsymbol{X}), exp\left\{\frac{-\epsilon}{\Delta M_t T}\right\}\right), \\
M_t(\boldsymbol{X})|\boldsymbol{\theta} &\overset{ind}{\sim} \text{Multinomial}_{d_t}(n, P_t(\boldsymbol{\theta})),\ t = 1, \ldots, T, \\
\boldsymbol{\theta} &\sim p_{\boldsymbol{\theta}}.
\end{aligned}
$$

- We use MCMC algorithms to sample from $\boldsymbol{\theta}|\mathcal{M}(\boldsymbol{X})$.

- Inferences are performed using $(P_1(\boldsymbol{\theta}), \ldots, P_T(\boldsymbol{\theta}))\,|\mathcal{M}(\boldsymbol{X})$.

Bayesian modeling approach and inference

- Instead of using $M(\boldsymbol{X})|\mathcal{M}(\boldsymbol{X})$, we use $M(\boldsymbol{X}^S)|\mathcal{M}(\boldsymbol{X})$.

- To make inferences about the confidential summary, we use

$$Pr(X_{(n+1)1} = c_1, \ldots, X_{(n+1)p} = c_p|\mathcal{M}(\boldsymbol{X})) =$$
$$\int Pr(X_{(n+1)1} = c_1, \ldots, X_{(n+1)p} = c_p|\boldsymbol{\theta})Pr(\boldsymbol{\theta}|\mathcal{M}(\boldsymbol{X}))d\boldsymbol{\theta}$$

to generate synthetic datasets $\boldsymbol{X}^S$ and induce a distribution via $\boldsymbol{X}^S \mapsto M(\boldsymbol{X}^S)$.

Illustrations with ACS PUMS

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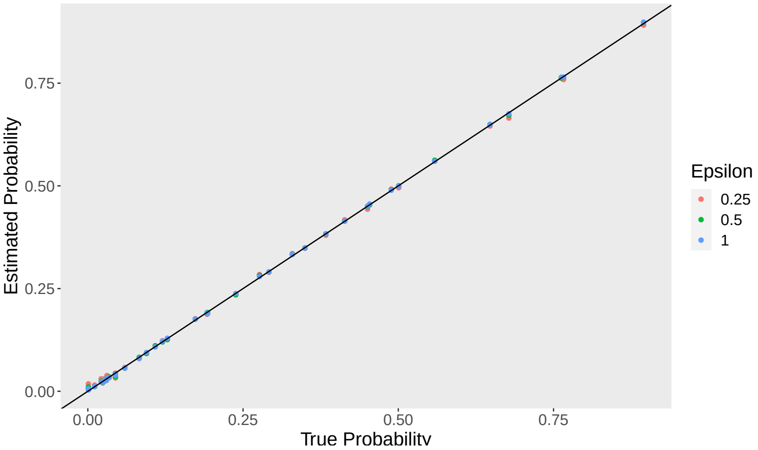
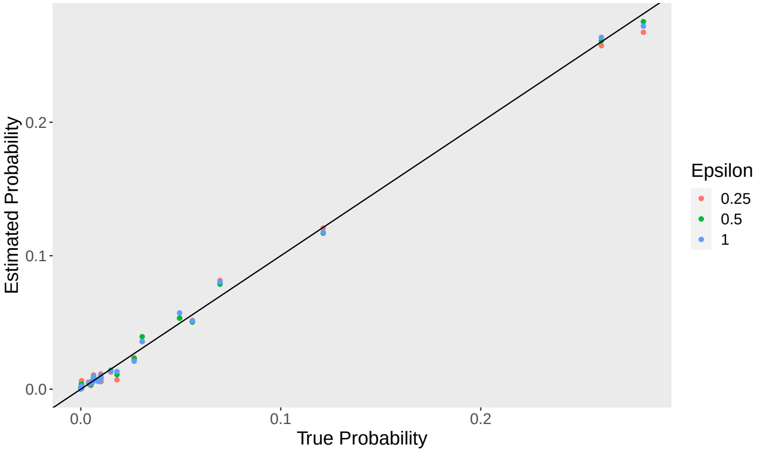
Illustrations with ACS PUMS

- True versus estimated two-way marginal tables.



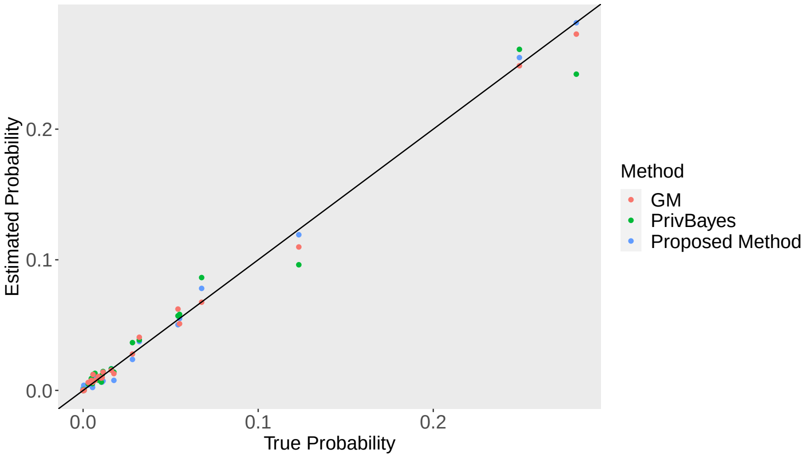
Illustrations with ACS PUMS

- True versus estimated full table.



Comparisons with existing methods

- True versus estimated full table ($\epsilon = 0.5$).



Concluding remarks

- We present a novel method to create differentially private synthetic data for contingency tables based on marginal counts.
- The simulation results indicate that our approach preserves the summaries.
- The proposed approach is complementary to existing releasing mechanisms.
- Our general strategy can be extended to more complex data structures.

Thank you!