Sample-based calibration of multiple surveys

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Outline

1. Introduction: recreational fishing surveys
2. Sample-based calibration
3. Application
4. Conclusions

Marine Recreational Information Program (MRIP)

- NOAA Fisheries is responsible for managing marine fisheries under the Magnuson-Stevens Act
  - MRIP produces estimates of marine recreational catch in US waters
  - input into stock assessment models, used to set annual catch limits
- In MRIP, multiple surveys are combined to create estimates
  - Access Point Angler Intercept Survey (APAIS)
  - Fishing Effort Survey (FES)
  - (others)

- APAIS: stratified multi-stage sample of fishing trips, collecting detailed data on trip and catch characteristics

- FES: stratified sample of general population households, collecting data on number of fishing trips over past 2 months

MRIP Estimation

- Combination of APAIS and FES:
  - survey weights in APAIS are calibrated to FES-obtained estimates of number of trips by state and wave
  - other adjustments for undercoverage of respective frames
- NOAA Fisheries provides public-use datasets with trip-level data and calibrated weights
- Variance estimation: current method uses linearization based on APAIS design
  - does not account for calibration to FES

Sample-based Calibration

- Calibration reduces the variance of survey estimators, so it is generally beneficial to account for it in variance estimation
  - In particular, variance of estimated control totals is zero (for population-based controls)
- But: *sample-based* calibration equalizes estimates between surveys, may not reduce variance
  - Important to account for variance contributions from both surveys into final variance estimates
- We describe methods to incorporate calibration into replicate variance estimation, when calibration totals are themselves random

2. Methodology: Primary Survey (APAIS)

- Sample s, weights w_i
- Population total $t_y = \sum_U y_i$ estimated by $\widehat{t}_y = \sum_s w_i y_i$
  - e.g. $\widehat{t}_y$ = estimated total catch of striped bass by private boat in GA during May-June 2019
- Replication variance estimator

$$\widehat{V}(\widehat{t}_y) = A \sum_{r=1}^{R} \left(\widehat{t}_y^{(r)} - \widehat{t}_y \right)^2$$

with $\widehat{t}_y^{(r)} = \sum_s w_i^{(r)} y_i$

- Replicate weights $w_i^{(r)}, r = 1, \ldots, R$ and constant A determined by replication method

  ⇒ Balanced Repeated Replication (BRR) with Fay's adjustment

  ⇒ $R = 160$

Calibration Survey (FES)

- Sample s_C, weights w_{Ci}
- Estimator $\widehat{\boldsymbol{t}}_{Cx} = \sum_{s_C} w_{Ci} \boldsymbol{x}_i$ of length H, to be used as controls
  - e.g. $\widehat{t}_{Cx,h} =$ estimated number of angler trips by private boat in GA during May-June 2019
- Estimator $\widehat{V}_C(\widehat{\boldsymbol{t}}_{Cx})$ of $H \times H$ variance-covariance matrix $\mathrm{Var}(\widehat{\boldsymbol{t}}_{Cx})$

Calibration of Primary Survey: Regression Estimation

- Regression estimator with calibration vector $\widehat{\boldsymbol{t}}_{Cx}$

$$\widehat{t}_{y,\mathrm{reg}} = \widehat{t}_y + (\widehat{\boldsymbol{t}}_{Cx} - \widehat{\boldsymbol{t}}_x)^T \widehat{\boldsymbol{\beta}} = \sum_s w_i^* y_i$$

- Define $e_i = y_i - \boldsymbol{\beta}_U^T \boldsymbol{x}_i$, then asymptotic variance

$$\begin{aligned} \mathrm{AVar}(\widehat{t}_{y,\mathrm{reg}}) &= \mathrm{Var}(\widehat{t}_e) && \text{(variance with fixed controls)} \\ &\quad + \boldsymbol{\beta}_U^T \mathrm{Var}(\widehat{\boldsymbol{t}}_{Cx}) \boldsymbol{\beta}_U && \text{(effect of random controls)} \end{aligned}$$

- For fixed $\widehat{\boldsymbol{t}}_{Cx}$, $\mathrm{Var}(\widehat{t}_e)$ consistently estimated by

$$\widehat{V}(\widehat{t}_{y,\mathrm{reg}}) = A \sum_{r=1}^{R} \left(\widehat{t}_{y,\mathrm{reg}}^{(r)} - \widehat{t}_{y,\mathrm{reg}} \right)^2$$

with

$$\widehat{t}_{y,\mathrm{reg}}^{(r)} = \widehat{t}_y^{(r)} + (\widehat{\boldsymbol{t}}_{Cx} - \widehat{\boldsymbol{t}}_x^{(r)})^T \widehat{\boldsymbol{\beta}}^{(r)} = \sum_s w_i^{*(r)} y_i$$

("apply calibration to each replicate")

Approaches to Estimate $\mathrm{AVar}(\widehat{t}_{y,\mathrm{reg}})$

1. Direct plug-in: $\widehat{V}(\widehat{t}_{\widehat{e}}) + \widehat{\boldsymbol{\beta}}^T \widehat{V}(\widehat{\boldsymbol{t}}_{Cx})\widehat{\boldsymbol{\beta}}$
2. Opsomer and Erciulescu (2021): when replicates are available for both surveys, create replicated control totals $\widehat{\boldsymbol{t}}_{Cx}^{(r)}$ to calibrate primary survey replicates (originally proposed by Kott (2005))
3. Fuller (1998): compute *eigen-decomposition* of $\widehat{V}(\widehat{\boldsymbol{t}}_{Cx})$ and perturb controls of primary survey replicates

Implementing Opsomer and Erciulescu (2021) Method

- Applicable when control survey has replicates for variance estimation
- Estimate vector $\widehat{\boldsymbol{t}}_{Cx} = \sum_{s_C} w_{Ci}\boldsymbol{x}_i$
- Replicate variance-covariance matrix estimator

$$\widehat{V}_C(\widehat{\boldsymbol{t}}_{Cx}) = A_C \sum_{r=1}^{R_C} \left(\widehat{\boldsymbol{t}}_{Cx}^{(r)} - \widehat{\boldsymbol{t}}_{Cx}\right)\left(\widehat{\boldsymbol{t}}_{Cx}^{(r)} - \widehat{\boldsymbol{t}}_{Cx}\right)^T$$

with $\widehat{\boldsymbol{t}}_{Cx}^{(r)} = \sum_{s_C} w_{Ci}^{(r)}\boldsymbol{x}_i$

- Replicate weights $w_{Ci}^{(r)}, r = 1, \ldots, R_C$ and constant A_C determined by control survey replication method
- Assume $R_C = R$

Implementing Opsomer and Erciulescu (2021) Method (2)

- Adjust control totals in replicates of primary survey, based on replicates from control survey

$$\begin{aligned}\widehat{t}^{(r)}_{y,\mathrm{reg}} &= \widehat{t}^{(r)}_{y} + \left(\widehat{\boldsymbol{t}}_{Cx} + a_r\left(\widehat{\boldsymbol{t}}^{(r)}_{Cx} - \widehat{\boldsymbol{t}}_{Cx}\right) - \widehat{\boldsymbol{t}}^{(r)}_{x}\right)^T \widehat{\boldsymbol{\beta}}^{(r)} \\ &= \widehat{t}^{(r)}_{y} + \left(\widehat{\boldsymbol{t}}_{Cx} - \widehat{\boldsymbol{t}}^{(r)}_{x}\right)^T \widehat{\boldsymbol{\beta}}^{(r)} + a_r\left(\widehat{\boldsymbol{t}}^{(r)}_{Cx} - \widehat{\boldsymbol{t}}_{Cx}\right)^T \widehat{\boldsymbol{\beta}}^{(r)}\end{aligned}$$

- Set $a_r = \sqrt{A_C/A}$, then

$$\widehat{V}(\widehat{t}_{y,\mathrm{reg}}) = A \sum_{r=1}^{R} \left(\widehat{t}^{(r)}_{y,\mathrm{reg}} - \widehat{t}_{y,\mathrm{reg}}\right)^2$$

consistent for

$$\mathrm{AVar}(\widehat{t}_{y,\mathrm{reg}}) = \mathrm{Var}(\widehat{t}_e) + \boldsymbol{\beta}_U^T \mathrm{Var}(\widehat{\boldsymbol{t}}_{Cx}) \boldsymbol{\beta}_U$$

Implementing Fuller (1998) Method

- Assume $H = R$ for now
- Compute eigen-decomposition of $\widehat{V}(\widehat{\boldsymbol{t}}_{Cx})$

$$\widehat{V}(\widehat{\boldsymbol{t}}_{Cx}) = \sum_{h=1}^{H} \lambda_h \boldsymbol{q}_h \boldsymbol{q}_h^T = \sum_{h=1}^{H} \boldsymbol{\delta}_h \boldsymbol{\delta}_h^T$$

- Adjust control totals in replicates of primary survey

$$\begin{aligned} \widehat{t}_{y,\mathrm{reg}}^{(r)} &= \widehat{t}_y^{(r)} + (\widehat{\boldsymbol{t}}_{Cx} + a_r \boldsymbol{\delta}_r - \widehat{\boldsymbol{t}}_x^{(r)})^T \widehat{\boldsymbol{\beta}}^{(r)} \\ &= \widehat{t}_y^{(r)} + (\widehat{\boldsymbol{t}}_{Cx} - \widehat{\boldsymbol{t}}_x^{(r)})^T \widehat{\boldsymbol{\beta}}^{(r)} + a_r \boldsymbol{\delta}_r \widehat{\boldsymbol{\beta}}^{(r)} \end{aligned}$$

- Set $a_r = 1/\sqrt{A}$, then

$$\widehat{V}(\widehat{t}_{y,\mathrm{reg}}) = A \sum_{r=1}^{R} \left(\widehat{t}_{y,\mathrm{reg}}^{(r)} - \widehat{t}_{y,\mathrm{reg}} \right)^2$$

consistent for

$$\mathrm{AVar}(\widehat{t}_{y,\mathrm{reg}}) = \mathrm{Var}(\widehat{t}_e) + \boldsymbol{\beta}_U^T \mathrm{Var}(\widehat{\boldsymbol{t}}_{Cx}) \boldsymbol{\beta}_U$$

Implementing Fuller (1998) Method (2)

- What if the numbers of control totals and replicates differ?
  - If $H \leq R$:

$$a_r = \begin{cases} \frac{1}{\sqrt{A}} & r = 1, \ldots, H \\ 0 & r = H+1, \ldots, R \end{cases}$$

  - If $H > R$: use $\boldsymbol{\delta}_h$ corresponding to R largest eigenvalues, which assumes that

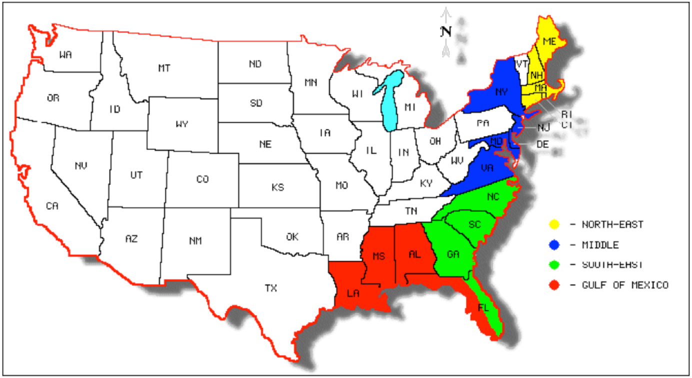
$$\sum_{r=1}^{R} \boldsymbol{\delta}_h \boldsymbol{\delta}_h^T \approx \widehat{V}(\widehat{\boldsymbol{t}}_{Cx})$$

    (low-rank approximation)
- Works for calibration by regression, post-stratification and raking

3. Application to APAIS Calibration

- 2019 APAIS and FES datasets
- Post-stratify APAIS weights to match FES estimated trip totals for 16 states, 2 modes (shore, private boat), 6 waves

  $\Rightarrow$ 160 control estimates ($R = H$)



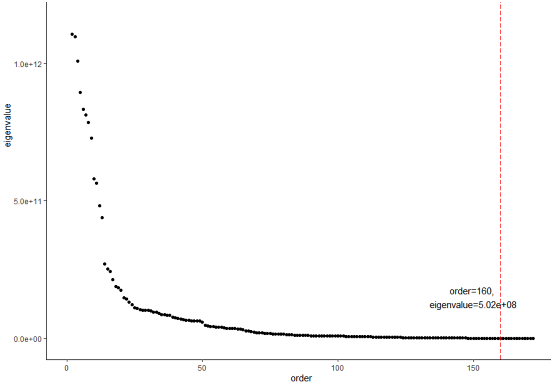
3. Application to APAIS Calibration (2)

- Distribution of CVs over 160 calibration domains

		Mean	Min.	1st Qu.	Median	3rd Qu.	Max.
Trips	FES	0.203	0.075	0.169	0.194	0.225	0.702
	APAIS (new)	0.203	0.075	0.168	0.193	0.225	0.700
	APAIS (old)	0.246	0.070	0.156	0.219	0.294	0.791
Red Snapper	APAIS (new)	0.620	0.201	0.462	0.654	0.746	1.097
	APAIS (old)	0.607	0.184	0.453	0.541	0.877	1.000

3. Application to APAIS Calibration (3)

- Investigate scenario when $H > R$
- 172 FES controls: estimated trip totals for 17 "states," 2 modes (shore, private boat), 6 waves
- Use $\boldsymbol{\delta}_h$ of 160 largest eigenvalues of $\widehat{V}(\widehat{\boldsymbol{t}}_{Cx})$



3. Application to APAIS Calibration (4)

- Distribution of CVs over 172 calibration domains

		Mean	Min.	1st Qu.	Median	3rd Qu.	Max.
	FES	0.208	0.100	0.171	0.196	0.233	0.702
Trips	APAIS (new)	0.207	0.100	0.171	0.195	0.233	0.700
	APAIS (old)	0.247	0.081	0.159	0.219	0.291	0.791
Red	APAIS (new)	0.631	0.199	0.498	0.633	0.791	1.073
Snapper	APAIS (old)	0.607	0.184	0.453	0.541	0.877	1.000

- Distribution of CVs over 160 calibration domains

		Mean	Min.	1st Qu.	Median	3rd Qu.	Max.
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Snapper	APAIS (old)	0.607	0.184	0.453	0.541	0.877	1.000

4. Conclusions

- Sample-based calibration can be very useful in practice, e.g.
  - organization conducts multiple surveys and wishes to report consistent estimates
  - following changes in survey methodology, survey results are no longer comparable with previous surveys and need to be adjusted
  - fixed controls are not available
  - multi-phase samples
- Important to reflect calibration to sample-based controls in measures of precision
- Can be accomplished easily within replication methods for primary survey

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