Designing Efficient Samples While Accounting for Anticipated Response Rates

Jonathan Mendelson[1] and Michael Elliott[2]

[1] U.S. Bureau of Labor Statistics

[2] University of Michigan

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Bottom line up front

- Efficient samples maximize precision for fixed data collection costs or minimize costs for fixed precision
- Survey designers have long used Neyman allocation (1934) and its extension, optimal allocation, to design efficient samples
  - Existing theory assumes complete response
  - Very little work considers efficient allocation under nonresponse
- We derive optimal allocations under nonresponse, observing efficiency gains of 25% when response rates vary highly by strata

Background concepts: stratified sample allocation

- Stratified random sampling (STSRS) designs are as follows:
  - Step 1. Divide population of N units into H *strata.*
  - Step 2. Within stratum h (for $h = 1,2, \ldots, H$), draw a simple random sample of n_h units from the N_h population units.
- *Sample allocation* here refers to choice of sample sizes, $\{n_h\}$.
- Under 100% response, the *optimal design* minimizes cost or (design-based) variance, holding the other constant.

Background concepts: Neyman allocation

- Neyman (1934) showed that the optimal STSRS allocation for estimating population means or totals is $n_h \propto N_h S_h$.
  - S_h is the stratum h standard deviation.
  - Assumes unit costs, c_h, are equivalent across strata, i.e., $c_h = c$.
- The optimal design under unequal costs is $n_h \propto N_h S_h / \sqrt{c_h}$.
- These results are hugely useful, and underlie many of today's probability samples, but assume 100% response rates.

Gap: how to handle nonresponse in STSRS allocation

- Existing optimal allocation theory (e.g., Stuart 1954; Cochran 1977) generally assumes complete response
  - Exception for dual-frame telephone surveys (Lohr & Brick, 2014)
  - Gap is evidenced by key sampling textbooks' lack of theoretical treatment for how to efficiently allocate samples while accounting for nonresponse
- Gap seems especially problematic given recent transitions toward self-administration and mixed-mode surveys (Olson et al., 2021)
  - Nonresponse can vary by subgroup and meaningfully affect costs

Our setup and notation

Category	Set of units (stratum h)	Number (stratum h)
Population units	U_h	N_h
Original sample (via STSRS)	$s_h \subset U_h$	n_h
Responding sample	$s_{Rh} \subset s_h$	r_h

- Estimate $\bar{Y}$ via $\hat{\bar{Y}} = \sum_{h=1}^{H} \frac{N_h}{N} \sum_{i \in s_{Rh}} \frac{y_{hi}}{r_h}$
  - $\hat{\bar{Y}}$ is a poststratified estimator under nonresponse
  - $\hat{\bar{Y}}$ arises by adjusting the (complete response) design-based estimator by the inverse of strata response rates

(Unconditional) variance of $\hat{\bar{Y}}$

- Assume stratum h units have the same response propensity, $\bar{\phi}_h$
  - Implies responding sample is conditionally STSRS
- Assume at least one respondent per stratum
  - Model as binomial with support for 0 removed
- Then $\mathrm{Var}\left(\hat{\bar{Y}}\right) = \sum_{h=1}^{H} \frac{N_h^2 S_h^2 \zeta_h(n_h, \bar{\phi}_h)}{N^2 n_h \bar{\phi}_h} - \frac{N_h S_h^2}{N^2}$

  where $\zeta_h(n_h, \bar{\phi}_h) := \mathrm{E}\left(\frac{1}{r_h}\right)\mathrm{E}(r_h)$ is a variance inflation factor that reflects the effect of the uncertainty in the responding sample sizes

We assume (variable) strata costs can be decomposed based on response status

- Ignoring fixed costs, we assume that total costs in stratum h are

$$C_h = r_h c_{R_h} + (n_h - r_h) c_{NR_h}, \text{ where}$$

c_{R_h} and c_{NR_h} denote unit costs for a single respondent or nonrespondent.

  - Let $\tau_h = c_{R_h} / c_{NR_h}$ denote the ratio of unit costs for resps. relative to nonresps.

- We consider a few scenarios:

Cost structure scenario	Assumptions	Expected cost per invitee
General (strata-specific)	$\{\tau_h\}$, $\{c_{NR_h}\}$ known	$c_h = c_{NR_h}(\bar{\phi}_h(\tau_h - 1) + 1)$
Common cost structure	$\tau_h = \tau;\ c_{NR_h} = c_{NR}$	$c_h = c_{NR}(\bar{\phi}_h(\tau - 1) + 1)$
Constant cost per invitee	$\tau_h = 1;\ c_{NR_h} = c_{NR}$	$c_h = c_{NR}$

We find the optimal allocation for minimizing the (unconditional) variance or expected costs

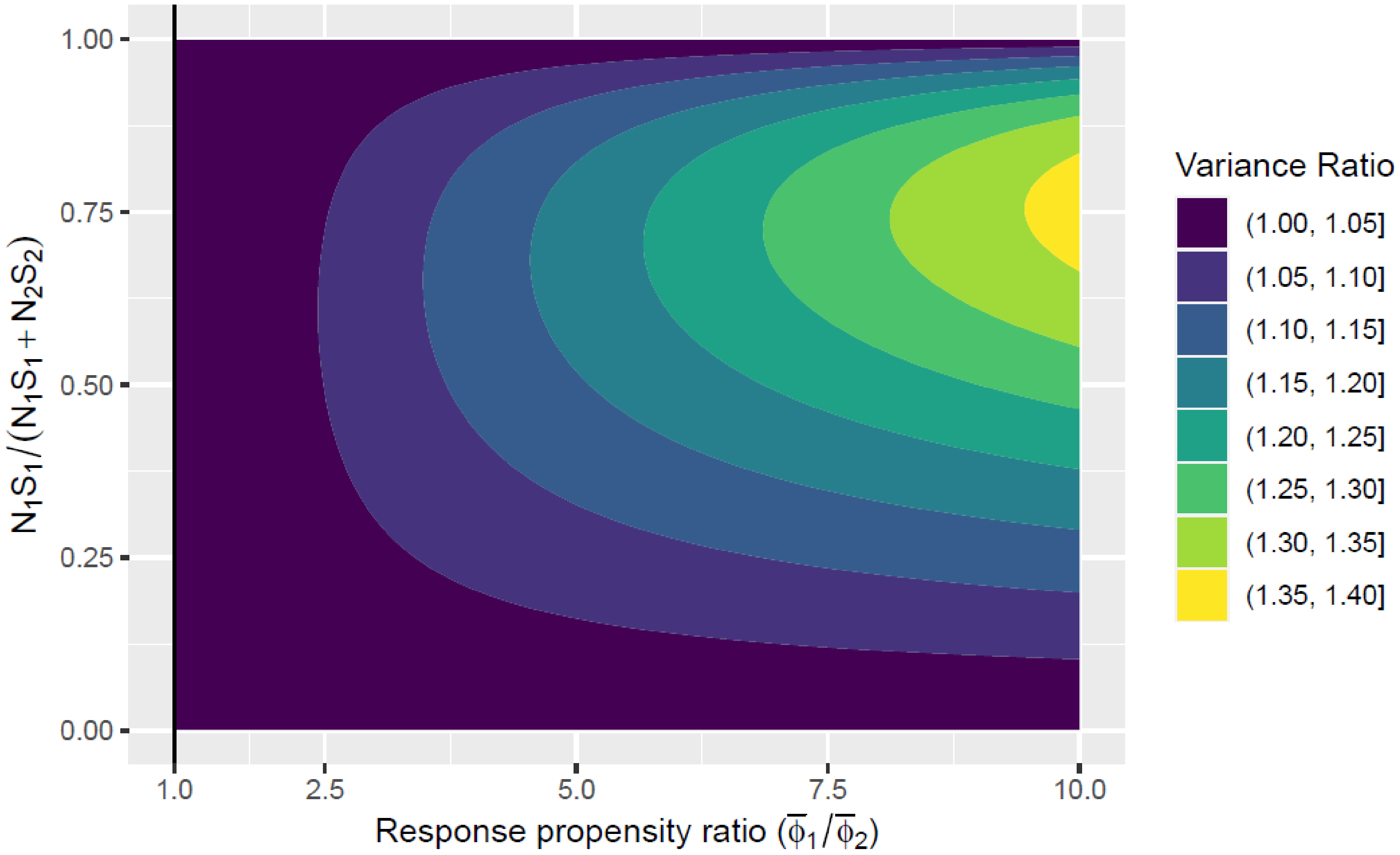
- $n_h \propto \frac{N_h S_h \sqrt{\zeta_h(n_h, \bar{\phi}_h)}}{\sqrt{\bar{\phi}_h c_h}}$
  - Note: n_h and c_h are defined re: invited sample
- Can compute $\{n_h\}$ iteratively by alternating between computations for $\{n_h\}$ and $\{\zeta_h(.)\}$
- We provide an R software implementation of the allocation with our JSSAM paper and via GitHub

We compare the (approximate) proposed allocation to two standard approaches:

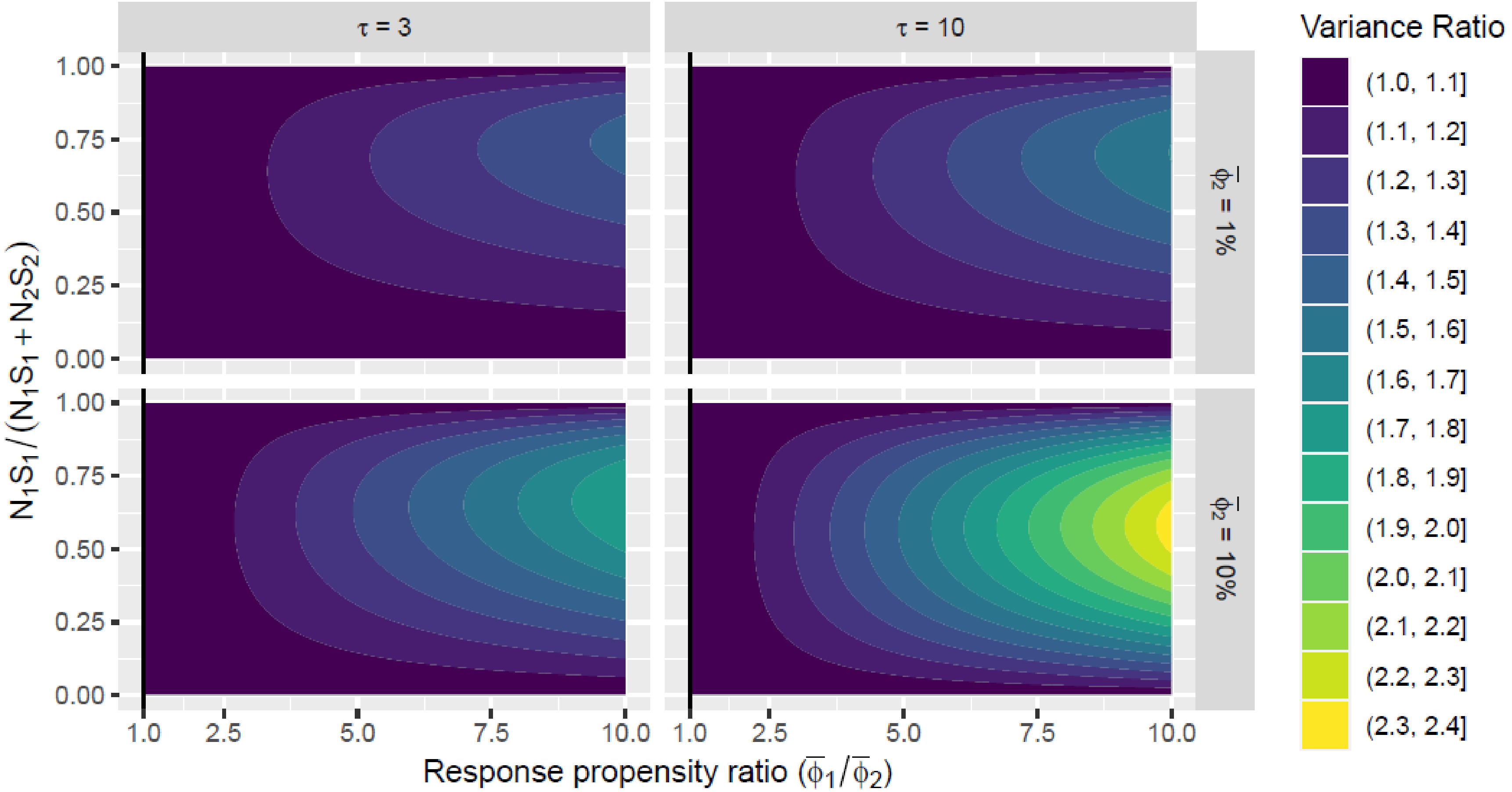
Allocation description	**Notation**	**Source of inefficiency**
Neyman/invitees	$n_h^{Ninv} \propto N_h S_h$	Excessive design effects
Neyman/respondents	$n_h^{Nresp} \propto N_h S_h / \bar{\phi}_h$	Excessive interview costs

- We consider the approximate variances of the standard approaches relative to that of the proposed design
  - We assume small r_h / N_h and large $n_h \bar{\phi}_h$
- We prove that under the *constant cost per invitee* scenario, Neyman allocations of invitees and respondents are equally inefficient!

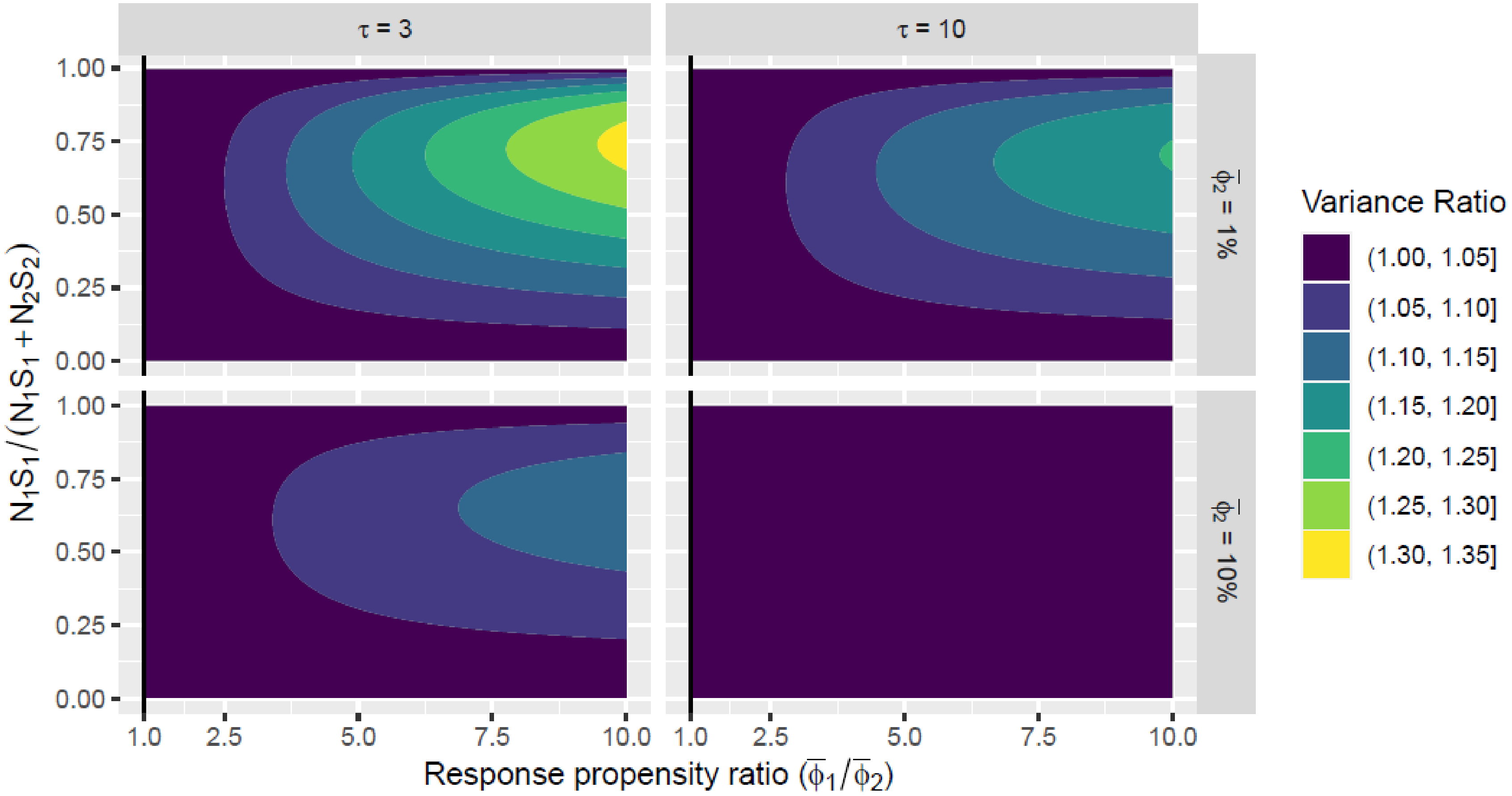
Variance ratio (VR) of standard approach (N/inv or N/resp) to proposed method under *common cost per invitee* ($H = 2$ example)



VR: Neyman/invitees to proposed method under *common cost structure* $(H = 2)$



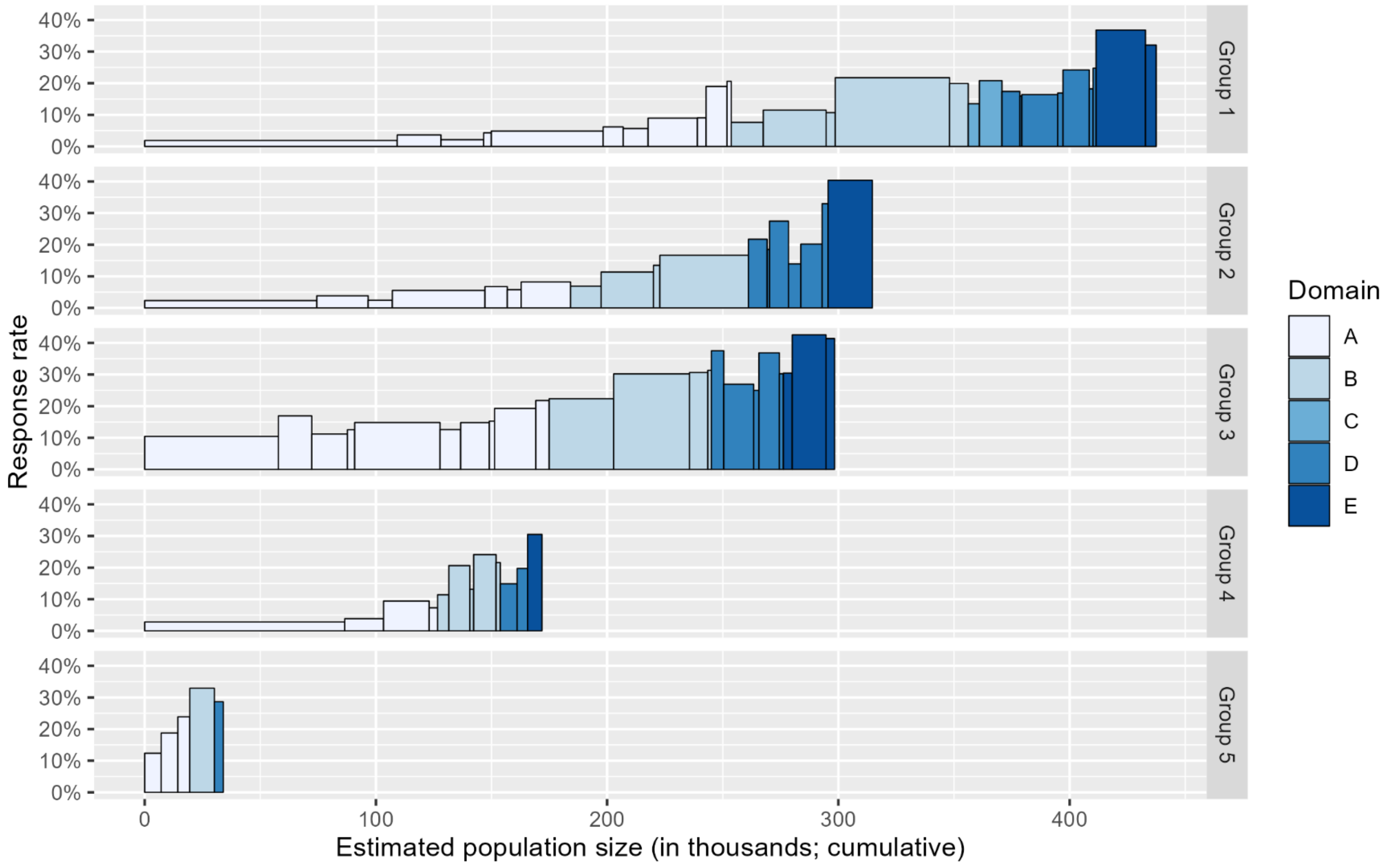
VR: Neyman/<u>respondents</u> to proposed method under *common cost structure* $(H = 2)$



Example. Application to Self-Administered Survey

- We compared allocations using a relevant public use dataset that was available at the time of conducting this research
- Survey characteristics
  - Sponsor type: federal agency (not a PSA)
  - Frame type: population list
  - Contact modes: mail and email (up to 4 mail contacts and 8 email contacts, for a total of 12)
  - Response mode: web
  - Sample analyzed: *n* = 75,548 invitees
    - Public data have base weights, poststrata, disposition codes
    - 14% response rate (AAPOR RR2)

Response rates varied substantially across subgroups



We allocated *n* = 50k under *constant cost per invitee*

Allocation	Notation	Response rate (%)	Respondents	$deff_w$	Effective respondents
Neyman/invitees	$n_h \propto N_h S_h$	13.6	6,794	2.37	2,865

Note. Assumes $r_h = n_h \bar{\phi}_h$ *and* S_h *constant across strata.*

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Proposed/approx.	$n_h \propto N_h S_h / \sqrt{c_h \bar{\phi}_h}$	9.0	4,494	1.26	3,570
Proposed/exact	$n_h \propto N_h S_h \sqrt{\zeta_h(.)} / \sqrt{c_h \bar{\phi}_h}$	9.0	4,496	1.26	3,570

Note. Assumes $r_h = n_h \bar{\phi}_h$ *and* S_h *constant across strata.*

- The proposed allocation increased the effective (responding) sample size by 25%, under equivalent total costs

We saw similar gains when allocating for specific Y's

- The Neyman-type allocations had variances 21%–26% higher than the proposed method

$100{,}000 * \mathrm{Var}\left(\hat{\bar{Y}}\right)$ for specific Y's, by allocation				
Allocation	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{Y}_4$
Neyman/invitees	1.60	1.50	1.61	1.41
Neyman/respondents	1.63	1.51	1.62	1.42
Proposed/approx.	1.32	1.20	1.29	1.12
Proposed/exact	1.32	1.20	1.29	1.12

Note. Assumes constant cost per invitee, $S_h = \hat{S}_h$, and $n = 50{,}000$.

Summary of results

- We extended classic theory for STSRS optimal allocation to allow for nonresponse
  - Our allocation strikes a better balance between design effects and cost-per-complete than existing practices
- We see the best gains when response rates vary greatly by strata
- N/inv and N/resp are equally inefficient if $\tau = 1$
  - Larger τ mitigates inefficiencies of N/resp but exacerbates that of N/inv
- We show the importance of incorporating anticipated nonresponse into the cost model assumed for design

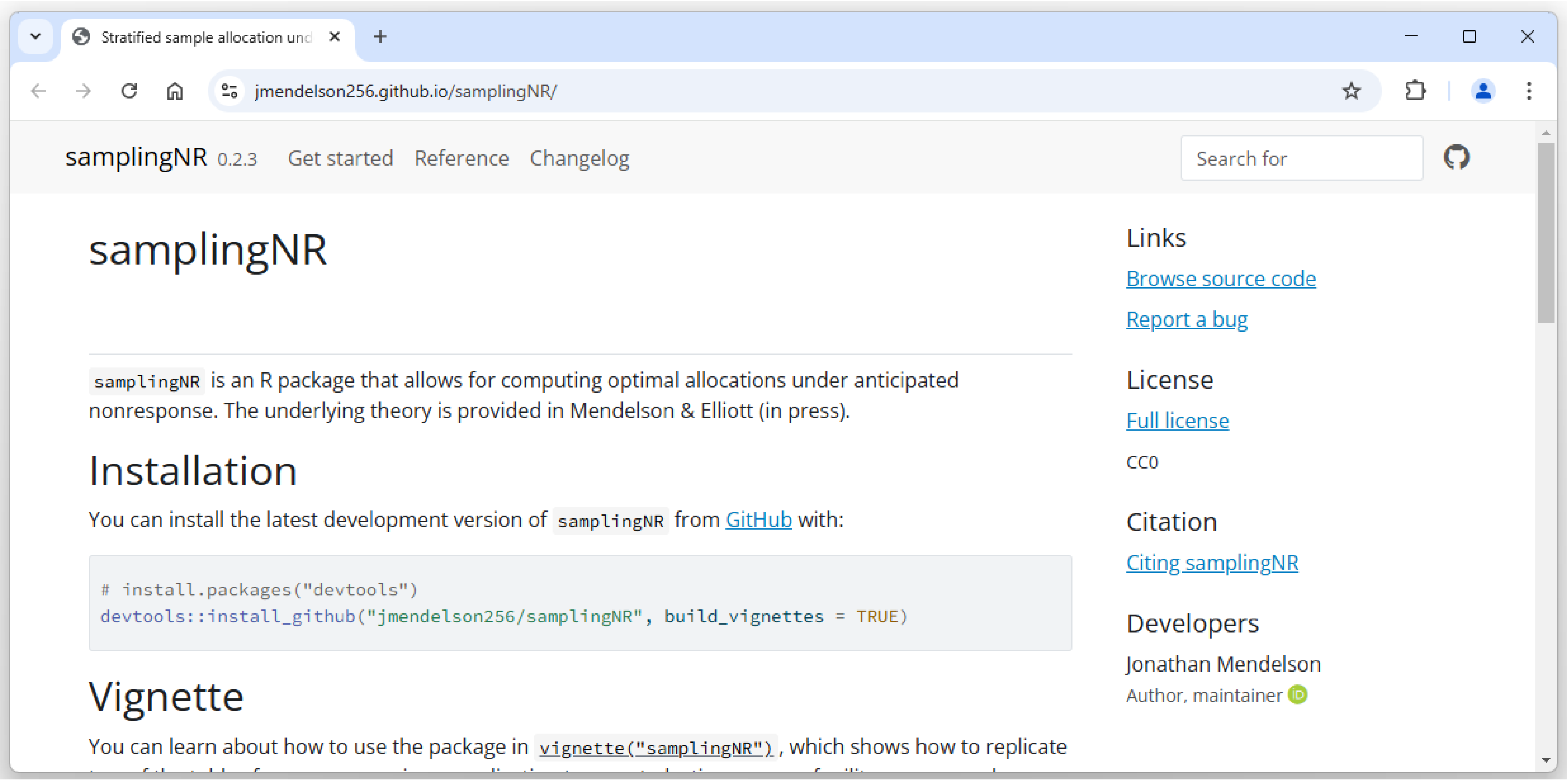
Limitations and future directions

- We assumed constant response propensities within strata, use of PS estimator under nonresponse, and known $\{\bar{\phi}_h\}$ and $\{S_h\}$
  - However, existing theory has similar assumptions but under 100% response!
  - Our allocation might be less sensitive to misspecified response rates than N/resp (due to milder oversampling of low response rate strata)
- Future work could consider different variance and/or cost structures
  - Could treat unknown eligibility via analogy to domain estimation
  - Likewise, more work needed on costs for other contexts (e.g., multi-stage, sequential mixed-mode)

Potential implications for statistical agencies

- Applicability is clearest for cross-sectional surveys with response rates that vary greatly by groups and in a manner driving costs
  - Provides another tool for dealing with response rate challenges, although potential gains will vary by survey
  - Note the potential trade-offs with domain (subgroup) precision
- Research illustrates utility of examining sampling assumptions, especially regarding costs and response rates

Software implementation in R is freely available at https://github.com/jmendelson256/samplingNR/



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Contact Information

Jonathan Mendelson
Research Statistician,
Office of Survey Research Methods
Behavioral Science Research Center
202-691-7268
mendelson.jonathan@bls.gov